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Fourier Transforms and Their Use in Electrical Circuit Analysis

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Abstract:

This paper investigates Fourier series and Fourier transforms as fundamental tools for electrical circuit analysis with an emphasis on bridging mathematical theory and practical applications. An analytical applied approach is adopted in which periodic and non-periodic signals are represented in the frequency domain to facilitate the analysis of circuit responses to various voltage and current excitations. The study presents the mathematical foundations of Fourier series and Fourier transforms, followed by the frequency domain analysis of RLC circuits and an examination of key phenomena associated with spectral analysis. To enhance practical insight, Mathcad was employed to generate graphical representations of the signals corresponding to each series and to demonstrate the effect of the number of harmonics on waveform reconstruction accuracy. The results demonstrate that Fourier based analysis, when supported by graphical visualization offers a clearer and more efficient interpretation of circuit behavior compared with conventional time-domain methods. Moreover, this approach provides a comprehensive framework for the design and analysis of electronic circuits, electrical filters, communication systems, and signal processing applications.

ملخص:

تتناول هذه الورقة البحثية متسلسلات فورييه وتحويلات فورييه بوصفها أدوات أساسية في تحليل الدوائر الكهربائية مع التركيز على الربط بين النظرية الرياضية والتطبيقات العملية .وقد تم اعتماد منهج تحليلي تطبيقي يتم من خلاله تمثيل الإشارات الدورية وغير الدورية في المجال الترددي بهدف تسهيل تحليل استجابة الدوائر لمختلف إثارات الجهد والتيار . وتعرض الدراسة الأسس الرياضية لمتسلسلات فورييه وتحويلات فورييه يلي ذلك تحليل الدوائر من نوع RLC في المجال الترددي إلى جانب مناقشة أهم الظواهر المرتبطة بالتحليل الطيفي. ولتعزيز الجانب العملي تم استخدام برنامج Mathcad لتوليد تمثيلات بيانية للإشارات المقابلة لكل متسلسلة وبيان تأثير عدد التوافقيات على دقة إعادة بناء الموجة. وتُظهر النتائج أن التحليل المعتمد على فورييه عند دعمه بالتمثيل البياني يوفر فهماً أوضح وأكثر كفاءة لسلوك الدوائر الكهربائية مقارنةً بالأساليب التقليدية في المجال الزمني. علاوة على ذلك يقدّم هذا النهج إطاراً متكاملاً لتصميم وتحليل الدوائر الإلكترونية، والمرشحات الكهربائية، وأنظمة الاتصالات، وتطبيقات معالجة الإشارات.

Keywords:

Fourier Transform ،Electrical Circuits Analysis، RLC Circuits ،Signal Representation ،Mathcad.

Introduction:

Signal and electrical system analysis constitutes one of the fundamental pillars of electrical engineering and applied physics as the efficiency of electrical circuit design and the understanding of their dynamic behavior largely depend on the ability to accurately characterize signals in both the time and frequency domains [3,5]. In this context Fourier series and Fourier transforms represent essential mathematical tools that enable the representation of periodic and non-periodic signals[4,7], respectively in the frequency domain thereby facilitating the analysis of linear time-invariant electrical circuits, Fourier series are based on the principle of decomposing periodic signals into an infinite sum of sinusoidal and cosinusoidal functions with multiple frequencies which allows for the investigation of electrical circuit responses to repetitive excitations such as alternating voltage and current waveforms. Fourier transforms on the other hand may be regarded as a natural generalization of Fourier series, providing an efficient mathematical framework for analyzing non-periodic signals that commonly arise in modern electronic systems [1]. The transformation from the time domain to the frequency domain using Fourier techniques has enabled the conversion of complex electrical circuit problems from differential equations into simpler algebraic relations, thereby facilitating the analysis of circuit responses to various sources and the investigation of phenomena such as resonance, attenuation, and signal distortion [2,6,8]. Moreover, this approach provides deeper insight into the influence of circuit elements, including resistors, capacitors, and inductors, on the frequency-domain behavior of electrical systems.

Research Objectives:

1. To examine Fourier series and Fourier transforms from both theoretical and practical perspectives, and to demonstrate their significance in electrical circuit analysis through the transformation from the time domain to the frequency domain.
2. To present the fundamental mathematical principles underlying Fourier series and Fourier transforms in a systematic and rigorous manner.
3. To analyze periodic electrical signals using Fourier series and to illustrate their representation in terms of discrete frequency components.
4. To clarify the relationship between time-domain and frequency-domain representations of electrical signals and to explain the transformation mechanisms between the two domains.
5. To investigate the frequency-domain response of basic electrical circuits (RLC circuits) subjected to various voltage and current excitations.

Research Problem:

Despite the significant importance of Fourier series and Fourier transforms in signal and electrical circuit analysis, their use is often presented in a fragmented manner, focusing either on abstract mathematical formulations or on engineering applications without a clear and systematic integration between the two. Moreover, there remains a need to clarify the practical role of frequency-domain analysis in simplifying electrical circuit models. Accordingly the research problem addressed in this study centers on a fundamental question regarding the extent to which Fourier series and Fourier transforms can provide an integrated analytical framework that effectively bridges mathematical foundations and practical applications in electrical circuit analysis.

Methodology:

This study employs an analytical applied methodology combining the theoretical mathematical framework of Fourier series and Fourier transforms with their practical application in the analysis of linear electrical circuits. The research is grounded in the mathematical modeling of circuits, focusing on the investigation of their responses in both the time and frequency domains. To support the analysis and enhance understanding, illustrative examples and analytical plots are used to demonstrate key concepts and the behavior of the circuits under various excitation conditions, and Mathcad software was employed to generate graphical representations of all signals**.**

1-Derivation of the Fundamental Mathematical Relations of Fourier Series

$$\int_{-L}^{L} \cos\frac{m\pi}{L}x \cos\frac{n\pi}{L}x\, dx = \begin{cases} 0, & m \neq n \\ L, & m = n \end{cases} \qquad (1a)$$

where the variables m and n take the values 1,2.3……

The following relations are used in trigonometry**:**

$$\cos A \cos B = 1/2\,[\cos(A-B) + \cos(A+B)] \qquad (1b)$$

Thus, in the case where $m \neq n$:

$$\int_{-L}^{L} \cos\frac{m\pi}{L}x\ \cos\frac{n\pi}{L}x\, dx = 1/2 \int_{-L}^{L} \left[\cos\frac{(m-n)\pi x}{L} + \cos\frac{(m+n)\pi x}{L}\right] dx$$

$$= \frac{L}{2(m-n)\pi}\sin\frac{(m-n)\pi x}{L}\Big|_{-L}^{L} + \frac{L}{2(m+n)\pi}\sin\frac{(m+n)\pi x}{L}\Big|_{-L}^{L} = 0 \qquad (1c)$$

Moreover $\cos^2 A = 1/2\,(1+\cos 2A)$ Consequently, in the case where $m = n$, we obtain**:**

$$\int_{-L}^{L} \cos\frac{m\pi}{L}x \cos\frac{n\pi}{L}x\, dx = 1/2 \int_{-L}^{L} \left(1 + \cos\frac{2n\pi x}{L}\right) dx$$

$$= 1/2\left[x + \frac{L}{2n\pi}\sin\frac{2n\pi}{L}\right]_{-L}^{L} = L \qquad (1d)$$

Similarly, it can be proven that**:**

$$\int_{-L}^{L} \sin\frac{m\pi x}{L}\ \sin\frac{n\pi x}{L}\ dx = \begin{cases} 0 & m \neq n \\ L & m = n \end{cases} \qquad (2a)$$

In trigonometry, we have $\sin A \sin B = 1/2\,[\cos(A-B) - \cos(A+B)]$,

$\sin^2 A = 1/2\,(1-\cos 2A)$. Thus, in the case where $m \neq n$, we obtain

$$\int_{-L}^{L} \sin\frac{m\pi x}{L}\sin\frac{n\pi x}{L}\ dx = 1/2 \int_{-L}^{L}\left[\cos\frac{(m-n)\pi x}{L} - \cos\frac{(m+n)\pi x}{L}\right] dx = 0 \qquad (2b)$$

While, in the case where $m = n$, we have

$$\int_{-L}^{L} \sin\frac{m\pi x}{L}\sin\frac{n\pi x}{L}\, dx = 1/2\int_{-L}^{L}\left(1-\cos\frac{2n\pi x}{L}\right)dx = L \qquad (2c)$$

It can be proven that $\int_{-L}^{L} \sin\frac{m\pi x}{L}\cos\frac{n\pi x}{L}\, dx = 0 \qquad (2d)$

$\because \quad \sin A \cos B = 1/2\,[\sin(A-B)+\sin(A+B)]$

In the case of $m \neq n$

$$\int_{-L}^{L} \sin\frac{m\pi x}{L}\cos\frac{n\pi x}{L}\, dx = 1/2\int_{-L}^{L}\left[\sin\frac{(m-n)\pi x}{L}+\sin\frac{(m+n)\pi x}{L}\right]dx = 0 \qquad (3a)$$

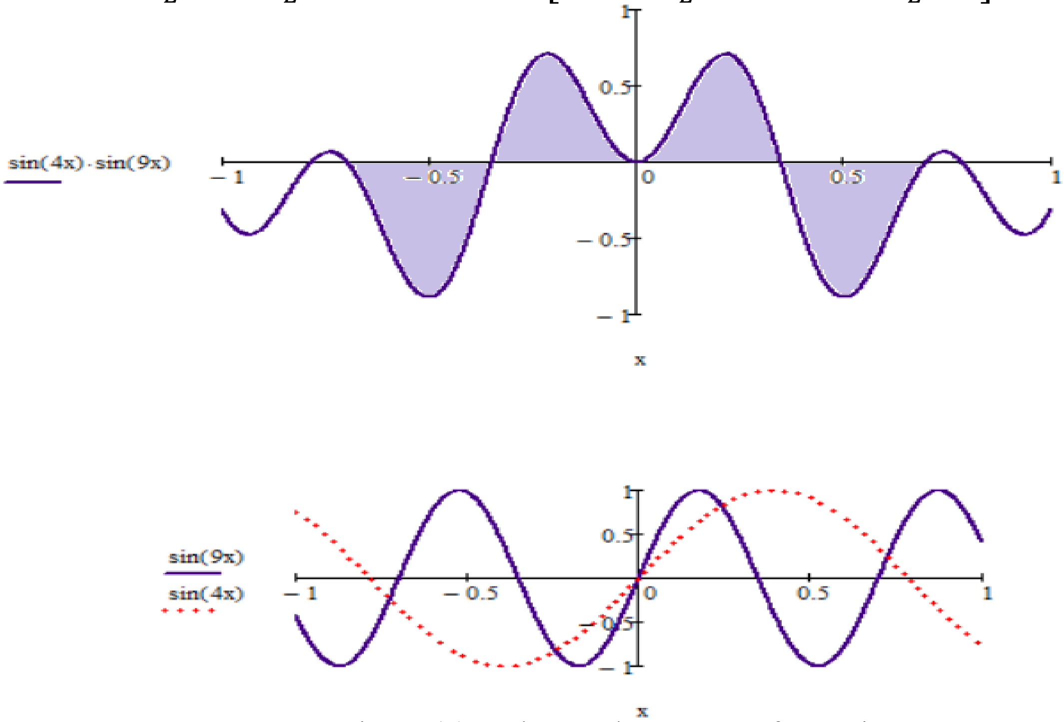

Figure (1) Orthogonal Systems Of Functions

Figure (1) illustrates the integral of the product of two functions, where the result of the integration is zero as shown in equation $(2b)$. The curves were plotted using Mathcad.

While, in the case where $m = n$ we have

$$\int_{-L}^{L} \sin\frac{m\pi x}{L}\cos\frac{n\pi x}{L}\, dx = 1/2\int_{-L}^{L}\sin\frac{2n\pi x}{L}\, dx = 0 \qquad (3b)$$

Assuming that the series

$$f(x) = a_0 + \sum_{n=1}^{\infty}\left(a_n\cos\frac{n\pi x}{L} + b_n\sin\frac{n\pi x}{L}\right) \qquad (4)$$

The value of a_n can be found by multiplying both sides of Equation (4) by $\cos\frac{m\pi x}{L}$, and integrating both sides from $-L$ to L**,** and by using Equations (1) and (3), we obtain**:**

$$\int_{-L}^{L} f(x)\cos\frac{m\pi x}{L}\,dx = \int_{-L}^{L} a_0 \cos\frac{m\pi x}{L} dx +$$

$$\sum_{n=1}^{\infty}\left(a_n \int_{-L}^{L} \cos\frac{m\pi x}{L}\cos\frac{n\pi x}{L} dx + b_n \int_{-L}^{L} \cos\frac{m\pi x}{L}\,\sin\frac{n\pi x}{L} dx\right) = a_m L$$

where $m \neq 0,$ and takes the values 1, 2, 3, …, and consequently:

$$a_m = \frac{1}{L}\int_{-L}^{L} f(x)\cos\frac{m\pi x}{L}\,dx \qquad (5a)$$

By multiplying both sides of Equation (4) by $\sin\frac{m\pi x}{L}$, and integrating from $-L$ to L, and by using Equations (2) and (3), we obtain:

$$\int_{-L}^{L} f(x)\sin\frac{m\pi x}{L}\,dx = \int_{-L}^{L} a_0 \sin\frac{m\pi x}{L} dx +$$

$$\sum_{n=1}^{\infty}\left(a_n \int_{-L}^{L} \sin\frac{m\pi x}{L}\cos\frac{n\pi x}{L}\,dx + b_n \int_{-L}^{L} \sin\frac{m\pi x}{L}\,\sin\frac{n\pi x}{L} dx\right) = b_m L$$

$$b_m = \frac{1}{L}\int_{-L}^{L} f(x)\sin\frac{m\pi x}{L}\,dx\,, m = 1,2,3, \dots .... \qquad (5b)$$

to find a_0 Integrating Equation (4) from $-L$ to L

$$\int_{-L}^{L} f(x)\,dx = \int_{-L}^{L} a_0\,dx + \int_{-L}^{L}\sum_{n=1}^{\infty}\left(a_n\cos\frac{n\pi x}{L} + b_n\sin\frac{n\pi x}{L}\right)dx$$

$$\therefore a_0 = \frac{1}{2L}\int_{-L}^{L} f(x)\,dx \qquad (5c)$$

According to Fourier's theorem any periodic function with frequency ω_0 can be expressed as follows:

$$f(t) = a_0 + a_1\cos\omega_0 t + b_1\sin\omega_0 t + a_2\cos 2\omega_0 t + b_2\sin 2\omega_0 t$$

$$a_3\cos 3\omega_0 t + b_3\sin 3\omega_0 t + \cdots \dots\dots\dots\dots$$

From Equation (4), the function $f(t)$ can be written as:

$$\therefore f(t) = a_0 + \sum_{n=1}^{\infty}(a_n\cos n\omega_0 t + b_n\ \sin n\omega_0 t) \qquad (6)$$

To determine the coefficients a_0، a_n ، b_n of the function $f(t)$, Equation (6) is integrated over one period, yielding:

$$\int_0^T f(t)\,dt = \int_0^T \left[a_0 + \sum_{n=1}^{\infty}(a_n\cos n\omega_0 t + b_n\,\mathrm{sinn}\,\omega_0\, t a_0)\right]dt$$

$$= \int_0^T a_0\,dt + \sum_{n=1}^{\infty}\left[\int_0^T a_n\cos n\omega_0\,t\,dt + \int_0^T b_n\sin n\omega_0\,t\,dt\right] \qquad (7)$$

$$\int_0^T f(t)\ dt = \int_0^T \mathrm{a}_0\, \mathrm{dt} = \mathrm{a}_0 T \Rightarrow \qquad \mathrm{a}_0 = \frac{1}{T}\int_0^T f(t)\ dt \tag{7a}$$

To find a_n we multiply both sides of Equation (6) by $\cos \mathrm{m}\omega_0 \mathrm{t}$, and integrate both sides:

$$\int_0^T f(t)\cos m\omega_0 \mathrm{t}\ dt = a_n \frac{T}{2}, m = n \Longrightarrow a_n = \frac{2}{T}\int_0^T f(t)\cos n\omega_0 \mathrm{t}\ dt \tag{7b}$$

Similarly we multiply both sides of Equation (6) by $\sin \mathrm{m}\omega_0 \mathrm{t}$, and integrate both sides:

$$b_n = \frac{2}{T}\int_0^T f(t)\cos n\omega_0 \mathrm{t}\ dt \tag{7c}$$

Example1: Find the Fourier series of the function with a period of 10

$$f(x) = \begin{cases} 0 & -5 < x < 0 \\ 3 & 0 < x < 5 \end{cases} \quad , \quad L = 5$$

The coefficients a_n و b_n و $a_0$are determined:

$$a_n = \frac{1}{L}\int_{-L}^{L} f(x)\cos\frac{n\pi x}{L}\ dx = \frac{1}{5}\int_{-5}^{5} f(x)\cos\frac{n\pi x}{5}\ dx$$

$$= \frac{1}{5}\left\{\int_{-5}^{0}(0)\cos\frac{n\pi x}{5}\ dx + \int_0^5 3\cos\frac{n\pi x}{5}\ dx\right\} = \frac{3}{5}\int_0^5 3\cos\frac{n\pi x}{5}\ dx$$

$$\therefore a_n = \frac{3}{5}\left(\frac{5}{n\pi}\sin\frac{n\pi x}{5}\right)\Big|_0^5 = 0\ , n \neq 0 \tag{8a}$$

$$b_n = \frac{1}{L}\int_{-L}^{L} f(x)\sin\frac{m\pi x}{L}\ dx = \frac{1}{5}\int_{-5}^{5} f(x)\sin\frac{n\pi x}{5}\ dx$$

$$= \frac{1}{5}\left\{\int_{-5}^{0}(0)\sin\frac{n\pi x}{5}\ dx + \int_0^5 3\sin\frac{n\pi x}{5}\ dx\right\} = \frac{1}{5}\int_0^5 3\sin\frac{n\pi x}{5}\ dx$$

$$\therefore b_n = \frac{3}{5}\left(\frac{-5}{n\pi}\cos\frac{n\pi x}{5}\right)\Big|_0^5 = \frac{3(1-\cos n\pi)}{n\pi} \tag{8b}$$

$$a_0 = \frac{1}{2L}\int_{-L}^{L} f(x)\ dx = \frac{1}{2(5)}\int_{-5}^{5} f(x)\ dx = \frac{1}{10}\int_{-0}^{5} 3\ dx = \frac{3}{2} \tag{8c}$$

By substituting Equations (8a, 8b, 8c) into Equation (4), the corresponding Fourier series becomes:

$$a_0 + \sum_{n=1}^{\infty}\left(a_n\cos\frac{n\pi x}{L} + b_n\sin\frac{n\pi x}{L}\right) = \frac{3}{2} + \sum_{n=1}^{\infty}\left(\frac{3(1-\cos n\pi)}{n\pi}\sin\frac{n\pi x}{5}\right)$$

$$= \frac{3}{2} + \frac{6}{\pi}\left(\sin\frac{\pi x}{5} + \frac{1}{3}\sin\frac{3\pi x}{5} + \frac{1}{5}\sin\frac{5\pi x}{5} + \frac{1}{7}\sin\frac{7\pi x}{5} + \cdots\right).$$

Example2: Expand the function $f(x) = x^2$, $-\pi < x < \pi$ into a Fourier series with a period of 2π

$$a_n = \frac{1}{\pi}\int_{-\pi}^{\pi} x^2 \cos n\pi x \ dx = \frac{1}{\pi}\left(x^2\frac{\sin nx}{n} + 2x\frac{\cos nx}{n^2} - 2\frac{\sin nx}{n^3}\right)\Big|_{-\pi}^{\pi}$$
$$= \frac{4}{n^2}\cos n\pi \ , n \neq 0$$

$$a_n = \frac{4}{n^2}(-1)^n \ = \begin{cases} \frac{4}{n^2} \ , & n \ even \\ \frac{-4}{n^2} \ , & n \ odd \end{cases}$$

$$b_n = \frac{1}{\pi}\int_{-\pi}^{\pi} x^2 \sin nx \ dx = \frac{1}{\pi}\left(x^2\frac{-\cos nx}{n} + 2x\frac{\sin nx}{n^2} + 2\frac{\cos nx}{n^3}\right)\Big|_{-\pi}^{\pi} = 0$$

$$a_0 = \frac{1}{2\pi}\int_{-\pi}^{\pi} x^2 \ dx = \frac{\pi^2}{3}$$

$$f(x) = \ a_0 + \sum_{n=1}^{\infty}\left(a_n \cos\frac{n\pi x}{L} + b_n \sin\frac{n\pi x}{L}\right)$$

$$= \frac{\pi^2}{3} - 4\left[\cos x - \ \frac{1}{2^2}\cos 2x + \frac{1}{3^2}\cos 3x - \frac{1}{4^2}\cos 4x + \cdots \ldots \ldots .\right]$$

$$or \ \ x^2 = \frac{\pi^2}{3} + \sum_{n=1}^{\infty}\left(\frac{4}{n^2}\cos n\pi \cos nx\right) \qquad (8d)$$

Now, we plot the Fourier series obtained in the previous example. This series converges to x^2 at all points of continuity within the domain $(\pi, -\pi)$. Since the Fourier series is periodic with a period of 2π, at the endpoints $\pm\,\pi$ and $n = 1,2,3,\ldots.$. the series must converge to the average of the right and left limits, which equals $\frac{\pi^2+\pi^2}{2} = \pi^2$ in each case. This yields Figure 2, which illustrates the desired plot.

Due to the continuity of the Fourier series at all endpoints, the result of this example can be extended to include these points as well.

$$\therefore \ x^2 = \frac{\pi^2}{3} + \sum_{n=1}^{\infty}\left(\frac{4}{n^2}\cos n\pi \cos nx\right) \qquad -\pi \leq x \leq \pi$$

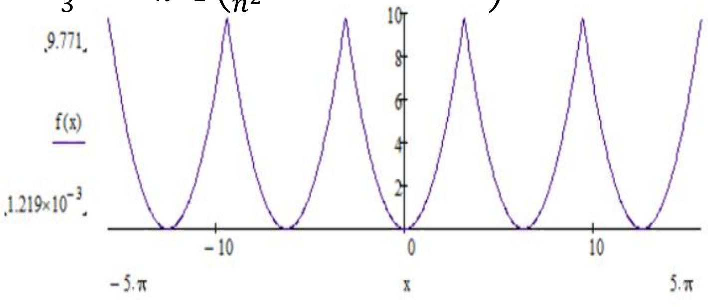

Figure (2)

Example3: $f(x) = x, -\pi < x < \pi$ with a period of 2π

$$a_n = \frac{1}{\pi}\int_{-\pi}^{\pi} x \cos n\pi x \ dx = 0 \ , \ a_0 = \frac{1}{2\pi}\int_{-\pi}^{\pi} x \ dx = 0 \qquad (9a)$$

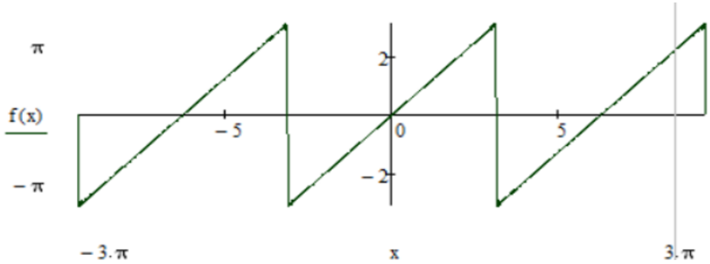

Figure (3)

$$b_n = \frac{1}{\pi}\int_{-\pi}^{\pi} x \sin nx \ dx = \frac{-1}{n}(2\cos n\pi) = \frac{-2}{n}(-1)^n \quad (9b)$$

$$f(x) = a_0 + \sum_{n=1}^{\infty}\left(a_n \cos\frac{n\pi x}{L} + b_n \sin\frac{n\pi x}{L}\right) = 2\left[\sin x - \frac{1}{2}\sin 2x + \frac{1}{3}\sin 3x - \cdots\right]$$

Example4: Find the Fourier series of the half-wave illustrated in the figure:

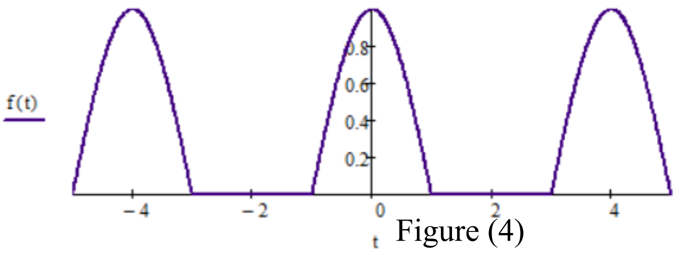

Figure (4)

This function is an even function, and therefore $b_n = 0$

$T=4$ ، $\omega_0 = \frac{2\pi}{T} = \frac{\pi}{2}$

$$f(t) = \begin{cases} 0, & -2 < t < -1 \\ \cos\frac{\pi}{2}t\ , & -1 < t < 1 \\ 0, & 1 < t < 2 \end{cases}$$

$$a_0 = \frac{2}{T}\int_0^{\frac{T}{2}} f(t)\ dt = \frac{2}{4}\left[\int_0^1 \cos\frac{\pi}{2}t\ dt + \int_1^2 0dt\right] = \frac{1}{2}\frac{2}{\pi}\sin\frac{\pi}{2}t\Big|_0^1 = \frac{1}{\pi} \quad (10a)$$

$$a_n = \frac{4}{T}\int_0^{\frac{T}{2}} f(t)\cos n\omega_0 \mathrm{t}\ dt = \frac{4}{4}\left[\int_0^1 \cos\frac{\pi}{2}t\ \cos\frac{n\pi}{2}t\ dt + 0\right]$$

By using equation (1b)

$$a_n = \frac{1}{2}\int_0^1 \left[\cos\frac{\pi}{2}(n+1)t + \cos\frac{\pi}{2}(n-1)t\ \right]dt$$

In the case of $n = 1$

$$a_1 = \frac{1}{2}\int_0^1 [\cos\pi t + 1]\,dt = \frac{1}{2}\left[\frac{\sin\pi t}{\pi} + t\right]\Big|_0^1 = \frac{1}{2} \quad (10b)$$

In the case of $n > 1$

$$a_n = \frac{1}{\pi(n+1)}\sin\frac{\pi}{2}(n+1) + \frac{1}{\pi(n-1)}\sin\frac{\pi}{2}(n-1) \qquad (10c)$$

The equation can be further simplified by considering the odd and even values of n.

In the case where n takes an odd value, both $(n+1)$, $(n-1)$ are even; therefore

$$\sin\frac{\pi}{2}(n+1) = 0 = \sin\frac{\pi}{2}(n-1)$$

In the case where n takes an even value, both $(n+1)$, $(n-1)$ are odd; therefore

$$\sin\frac{\pi}{2}(n+1) = -\sin\frac{\pi}{2}(n-1) = \cos\frac{n\pi}{2} = (-1)^{\frac{n}{2}}$$

$$a_n = (-1)^{\frac{n}{2}}\left(\frac{1}{\pi(n+1)} - \frac{1}{\pi(n-1)}\right) = \frac{-2(-1)^{\frac{n}{2}}}{\pi(n^2-1)} \ , \ \text{n} = \text{even} \qquad (10d)$$

$$f(t) = \frac{1}{\pi} + \frac{1}{2}\cos\frac{\pi}{2}t - \frac{2}{\pi}\sum_{n=even}^{\infty}\frac{(-1)^{\frac{n}{2}}}{(n^2-1)}\cos\frac{n\pi}{2}t \qquad (10e)$$

The series can be further simplified by using Equation $n = 2k, \quad k = 1,2,3,4, \ldots \ldots$

$$f(t) = \frac{1}{\pi} + \frac{1}{2}\cos\frac{\pi}{2}t - \frac{2}{\pi}\sum_{k=1}^{\infty}\frac{(-1)^{k}}{(4k^2-1)}\cos k\pi t \qquad (10e)$$

The Fourier series representation of the circuit shown in Figure (5), along with its associated signal can be obtained.

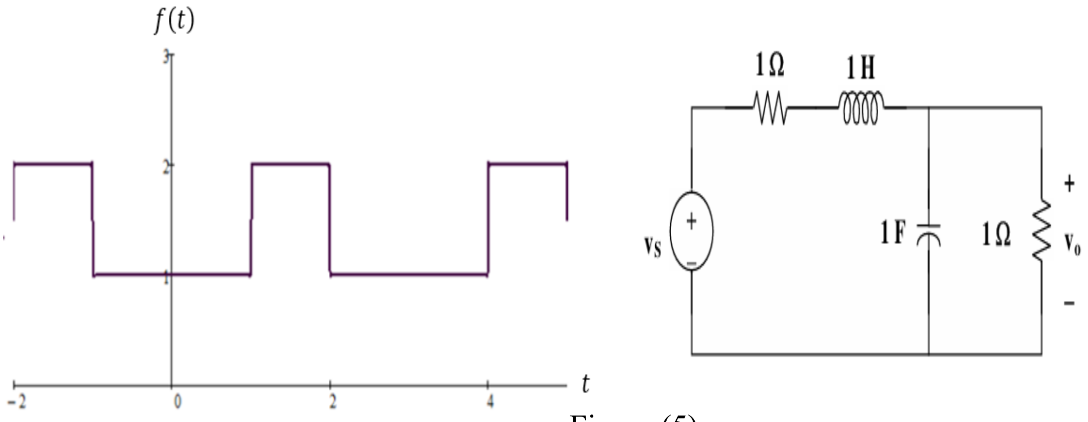

Figure (5)

Since the signal is even , hence $b_n = 0$

$$T = 3 \ ، \ \omega_0 = \frac{2\pi}{T} = \frac{2\pi}{3}$$

$$V_S(t) = \begin{cases} 1 \ , & 0 < t < 1 \\ 2 \ , & 1 < t < 1{,}5 \end{cases}$$

$$a_0 = \frac{2}{3}\left[\int_0^1 dt + \int_1^{1.5} 2dt\right] = \frac{4}{3} \quad (11a)$$

$$a_n = \frac{4}{3}\left[\int_0^1 \cos\left(\frac{2n\pi}{3}t\right)dt + \int_1^{1,5} 2\cos\left(\frac{2n\pi}{3}t\right)dt\right] \quad (11b)$$

$$= \frac{4}{3}\left[\frac{3}{2n\pi}\sin\left(\frac{2n\pi}{3}t\right)\Big|_0^1 + \frac{6}{2n\pi}\sin\left(\frac{2n\pi}{3}t\right)\Big|_1^{1.5}\right] = -\frac{2}{n\pi}\sin\left(\frac{2n\pi}{3}\right)$$

$$\therefore V_S(t) = \frac{4}{3} - \frac{2}{\pi}\sum_{n=1}^{\infty}\frac{1}{n}\sin\left(\frac{2n\pi}{3}\right)\cos\left(\frac{2n\pi}{3}t\right) \quad (11c)$$

2-Exponential Fourier Series:

The Fourier series in equation (6) can be written in exponential form by expressing the sine and cosine functions in exponential form using Euler's identity.

$$\sin n\omega_0 t = \frac{1}{2j}\left[e^{jn\omega_0 t} - e^{-jn\omega_0 t}\right], \quad \cos n\omega_0 t = \frac{1}{2}\left[e^{jn\omega_0 t} + e^{-jn\omega_0 t}\right] \quad (12a)$$

$$\therefore\ f(t) = a_0 + \frac{1}{2}\sum_{n=1}^{\infty}\left[(a_n - jb_n)e^{jn\omega_0 t} + (a_n + jb_n)e^{-jn\omega_0 t}\right] \quad (12b)$$

Substitution can be employed to simplify equation (12b) by introducing new coefficients as follows:

$$c_0 = a_0\ ,\ c_n = \frac{(a_n - jb_n)}{2}\ ,\quad c_{-n} = \frac{(a_n + jb_n)}{2} \quad (12c)$$

$$\therefore\ f(t) = c_0 + \sum_{n=1}^{\infty}\left[c_n e^{jn\omega_0 t} + c_{-n}e^{-jn\omega_0 t}\right] = \sum_{-\infty}^{\infty} c_n e^{jn\omega_0 t} \quad (12d)$$

This expression corresponds to the complex (or exponential) Fourier representation of the function $f(t)$.

The exponential Fourier series of the periodic function $f(t)$ describes the spectrum of the function in terms of the amplitudes and phase angles of the alternating current components at positive and negative harmonic frequencies, And the coefficients of the three Fourier series forms the sine, cosine form, the amplitude phase form, and the exponential form are related by the following relationships.

$$A_n \angle\emptyset_n = (a_n - jb_n) \quad (12e)$$

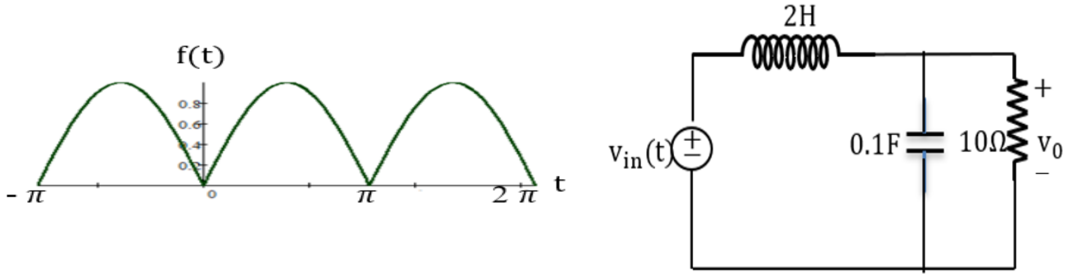

Figure (6)

$b_n = 0$ ، $T = \pi$ ، $\omega_0 = \frac{2\pi}{T} = \frac{2\pi}{\pi} = 2, \omega_n = n\omega_0 = 2n$

By applying equation (7a), we obtain: ($a_0 = \frac{\pi}{2}$), And by applying equation (7b), we obtain:

$$a_n = -\frac{4}{\pi}\frac{1}{4n^2-1}$$

$$\therefore v_{in}(t) = \left[\frac{\pi}{2} - \frac{4}{\pi}\sum_{n=1}^{\infty}\frac{1}{4n^2-1}\cos 2nt\right] volts$$

For the nth harmonic

$$v_{in} = -\frac{4}{\pi}\frac{1}{4n^2-1}\angle 0, \quad 0.1\ \text{F} = \frac{1}{j\omega_n C} = \frac{-5j}{n}, 2H = j\omega_n L = 4jn, Z = \frac{-10j}{(2n-j)}$$

$$v_0 = \left[\frac{Z}{(Z+4nj)}\right] v_{in} = \frac{-10jv_{in}}{(4+j(8n-10))} = -\frac{10j}{j(8n-10)}\left(-\frac{4\angle 0}{\pi(4n^2-1)}\right)$$

$$= \frac{40\angle\{90^\circ - \tan^{-1}(2n-2.5)\}}{\pi(4n^2-1)\sqrt{16+(8n-10)^2}}$$

$$v_0(t) = \left[\frac{2}{\pi} + \sum_{n=1}^{\infty} A_n \cos(2nt+\theta_n)\right] volts, A_n = \frac{20}{\pi(4n^2-1)\sqrt{16n^2-40n+29}},$$

$$\theta_n = 90^\circ - \tan^{-1}(2n-2.5)$$

3-Fourier Transform:

Fourier series are used to represent periodic functions as a sum of sinusoidal components and to extract their frequency spectra, whereas the Fourier transform extends this concept to non-periodic functions by treating them as periodic functions with an infinite period. The Fourier transform is an integral transform that maps a function from the time domain to the frequency domain and is distinguished by its ability to handle signals and circuits with inputs defined over the entire time interval $0 > t$, $t > 0$ in contrast to the Laplace transform which is restricted to $t > 0$, and requires initial conditions. The Fourier transform is derived from the Fourier series by allowing the period of the periodic function to approach infinity, resulting in a continuous spectrum rather than a discrete one. The Fourier transform is widely used in the analysis of electrical circuits, communication systems, and digital signal processing, and it is defined by the following expression:

$$f(t) = \frac{1}{2\pi}\int_{-\infty}^{\infty}\left[\int_{-\infty}^{\infty} f(t)\, e^{-j\omega t}\, dt\right] e^{j\omega t}\, d\omega \qquad (13)$$

The quantity enclosed in parentheses is defined as the Fourier transform of the function $f(t)$ and is expressed by $f(\omega)$.

$$F(\omega) = \mathcal{F}[f(t)] = \int_{-\infty}^{\infty} f(t)\, e^{-j\omega t}\, dt \qquad (14)$$

Where $\mathcal{F}$ denotes the Fourier transform operator, and the Fourier transform is an integral that converts the function $f(t)$ from the time domain to the frequency domain.

Equation (13) can be expressed as a function of (ω), and consequently the inverse Fourier transform is obtained as follows:

$$f(t) = \mathcal{F}^{-1}[F(\omega)] = \frac{1}{2\pi}\int_{-\infty}^{\infty} F(\omega)\, e^{j\omega t}\, d\omega \qquad (15)$$

In the following figure, the Fourier transform can be determined:

$$f(t) = \mathrm{e}^{-\mathrm{at}}u(t) = \begin{cases} \mathrm{e}^{-\mathrm{at}}, & t > 0 \\ 0, & t < 0 \end{cases}$$

$$f(\omega) = \int_{-\infty}^{\infty} f(t)\, e^{-j\omega t}\, dt = \int_{0}^{\infty} \mathrm{e}^{-\mathrm{at}}\, e^{-j\omega t} dt$$

$$= \frac{-1}{a+j\omega}\ \mathrm{e}^{-\mathrm{at}-\mathrm{j}\omega}\Big|_{0}^{\infty} = \frac{1}{a+j\omega}$$

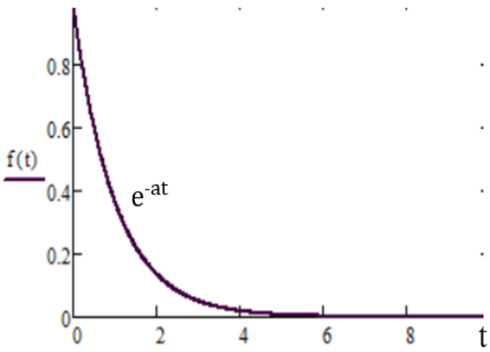

Figure (7)

Table (1) Fourier Transform Pairs:

$f(t)$	$F(\omega)$
1	$2\pi\delta(\omega)$
$\delta(t)$	1
$\mathrm{e}^{-\mathrm{at}}\, u(t)$	$\frac{1}{a + j\omega}$
$\mathrm{e}^{-\mathrm{at}}\, u(-t)$	$\frac{1}{a - j\omega}$
$u(t)$	$\pi\delta(\omega) + \frac{1}{j\omega}$
$\lvert t \rvert$	$\frac{-2}{\omega^2}$
$u(t+\tau)$- $u(t-\tau)$	$\frac{2}{\omega}\sin\omega\tau$
$\sin\omega_0 t$	$j\pi[\delta(\omega+\omega_0) - \delta(\omega-\omega_0)]$
$\cos\omega_0 t$	$\pi[\delta(\omega+\omega_0) + \delta(\omega-\omega_0)]$
$t^n \mathrm{e}^{-\mathrm{at}}\, u(t)$	$\frac{n:}{(a+j\omega)^{n+1}}$

$e^{-a\|t\|}$	$\frac{2a}{a^2+\omega^2}$
$e^{-at}\sin\omega_0 t\, u(t)$	$\frac{\omega_0}{(a+j\omega)^2+{\omega_0}^2}$
$e^{-at}\cos\omega_0 t\ u(t)$	$\frac{a+j\omega}{(a+j\omega)^2+{\omega_0}^2}$

Now, Fourier transforms can be applied to the following electrical circuit:

$$I_s(t) = 10\sin 2t,\ 0.5\ \mu F = \frac{1}{j\omega C} = \frac{2}{j\omega} \quad , \quad H(\omega) = \frac{I_0(\omega)}{I_s(\omega)}$$

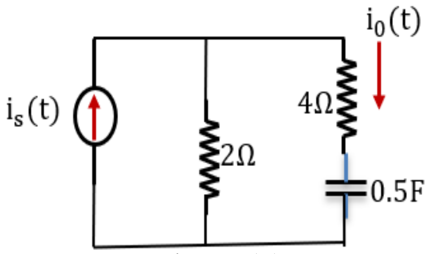

Figure (8)

By substituting the value of $\sin\omega_0 t$ from the table (1), we obtain:

$$I_s(\omega) = j\pi 10[\delta(\omega+2) - \delta(\omega-2)]$$

By current division RULE (**CDR**):

$$I_0(\omega) = \frac{I_s(\omega)}{2+4+\frac{2}{j\omega}} \times 2 \Rightarrow \qquad \frac{I_0(\omega)}{I_s(\omega)} = H(\omega) = \frac{j\omega}{1+3j\omega}$$

$$I_0(\omega) = I_s(\omega)H(\omega) = \frac{j\pi \times 10j\omega[\delta(\omega+2) - \delta(\omega-2)]}{1+3j\omega}$$

$$= \frac{-10\pi\omega[\delta(\omega+2) - \delta(\omega-2)]}{1+3j\omega} = \frac{10\pi\omega[\delta(\omega-2) - \delta(\omega+2)]}{1+3j\omega}$$

Since the inverse Fourier transform $I_0(\omega)$ cannot be obtained from Table (1), the inverse Fourier transform is evaluated using Equation (15). Consequently**:**

$$I_0(t) = \mathcal{F}^{-1}[I_0(\omega)] = \frac{10\pi}{2\pi}\left[\int_{-\infty}^{\infty} \frac{\omega\delta(\omega-2)}{1+3j\omega}\ e^{j\omega t}\ d\omega\ - \int_{-\infty}^{\infty} \frac{\omega\delta(\omega+2)}{1+3j\omega}\ e^{j\omega t}\ d\omega\right]$$

By using shifting property

$$\int_{-\infty}^{\infty} \delta(\omega-\omega_0)f(\omega)d\omega = f(\omega_0)$$

$$I_0(t) = \frac{10\pi}{2\pi}\left[\int_{-\infty}^{\infty} \frac{\delta(\omega-2)}{1+3j\omega}\ \omega e^{j\omega t}\, d\omega\ - \int_{-\infty}^{\infty} \frac{\delta(\omega+2)}{1+3j\omega}\ \omega e^{j\omega t}\, d\omega\right]$$

$$= \frac{10\pi}{2\pi}\left[\frac{2}{1+3\times 2j}\ e^{2jt} - \frac{-2}{1+3\times -2j}\ e^{-2jt}\right] = 10\left[\frac{e^{2jt}}{1+6j} + \frac{e^{-2jt}}{1-6j}\right]$$

$$= 10\left[\frac{e^{2jt}}{6.082\angle 80.54^{\circ}} + \frac{e^{-2jt}}{6.082\angle 80.54^{\circ}}\right] = 1.64\left[\ e^{2jt-80.54^{\circ}} +\ e^{-(2jt-80.54^{\circ})}\right]$$

$$\therefore I_0(t) = 3.28\cos(2t - 80.54^{\circ})A.$$

Results:

The results indicate that Fourier series and Fourier transforms serve as effective tools for the analysis of linear electrical circuits. The frequency domain representation enables precise characterization of signal components and provides a clear interpretation of circuit responses to both periodic and non-periodic signals. Frequency-domain analysis has contributed to simplifying the governing equations of RLC circuits, converting them into algebraic relations that are easier to handle while preserving the physical significance of the studied phenomena. Furthermore, the results highlight the importance of using Mathcad to support the mathematical analysis through graphical representation of signals, demonstrating the effect of the number of harmonics on approximation accuracy and enhancing the visual understanding of the relationship between mathematical formulation and practical circuit behavior. Comparisons with conventional time domain analysis confirm the superiority of Fourier-based analysis in studying complex signals and signal distortion, supporting its use as a comprehensive analytical framework for the design and analysis of electronic circuits and communication systems.

Conclusion:

This study concludes that Fourier series and Fourier transforms constitute effective mathematical tools for the analysis of linear electrical circuits, as frequency-domain analysis provides a more accurate understanding of electrical signal behavior and circuit responses compared with conventional time-domain analysis. The utilization of Mathcad for computational and graphical purposes further emphasizes the importance of integrating mathematical analysis with computational tools to clarify the practical behavior of electrical circuits. It is therefore concluded that Fourier-based methods offer a comprehensive analytical framework that can be reliably employed in the design and analysis of electronic circuits and communication systems, with the potential for future extensions to more advanced applications in signal and electrical system analysis.

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